AMBO IBSBS SCHOOL

Physics
G-12
Unit 2: Short Note

Unit 2 Two Dimensional Motion

Introduction!

Definition:

Two dimensional motion: is the motion of objects in a plane, involving both horizontal and vertical components.

Examples:

- ✓ *Kicked football*
- ✓ *Planets' orbits*
- ✓ *Bicycles rounding curves*
- ✓ *Car wheels' rotation*

Nature of Motion

- Curved Paths: most natural motions follow curved paths rather than straight lines.
- Importance: understanding curved motion is essential for analyzing real-world scenarios.

Note!

- Two-dimensional kinematics is crucial for understanding the curved paths of objects in nature.
- Specific types of two-dimensional motion: projectile motion and circular motion.

Projectile Motion: Understanding the Concept and Applications

What is Projectile Motion?

Definition: A projectile refers to an object that is in flight after being thrown or projected.

Examples:

- ✓ *a football kicked in a game*
- ✓ *a cannonball fired from a cannon*
- ✓ *a bullet fired from a gun*
- ✓ *the flight of a golf ball, and*
- ✓ *a jet of water escaping a hose.*

Which Motion is Different?

Options:

1. A ball thrown horizontally in the air.
2. A bullet fired from a gun.
3. A javelin thrown by an athlete.
4. A bird flying in the air.

Discussion Prompt: Which motion is different from the others? Why?

Projectile Motion: 1, 2 & 3

Non-Projectile Motion: 4

✓ *A bird flying in the air (actively powered flight, not a projectile*

Horizontal and Vertical Components

Horizontal Motion:

- ✓ Constant velocity (uniform motion) due to no horizontal acceleration (ignoring air resistance).

Vertical Motion:

- ✓ Accelerated motion (uniformly accelerated) due to gravity.

Common Variable: *Time (t) is the common variable in both horizontal and vertical motion analysis.*

Horizontal and Inclined Projectile Motions

i) Horizontal Projectile Motion

introduction to projectile motion.mp4

Definition: Horizontal projectile motion refers to the motion of an object that is projected horizontally from a certain height and moves under the influence of gravity alone.

Initial conditions:

- ✓ Initial vertical velocity (v_y) is zero.
- ✓ the projectile has only horizontal velocity at the beginning.
- ✓ the object has an initial velocity only in the horizontal direction.

Horizontal Motion:

- ✓ The horizontal velocity remains constant throughout the motion.
- ✓ Horizontal acceleration is zero.

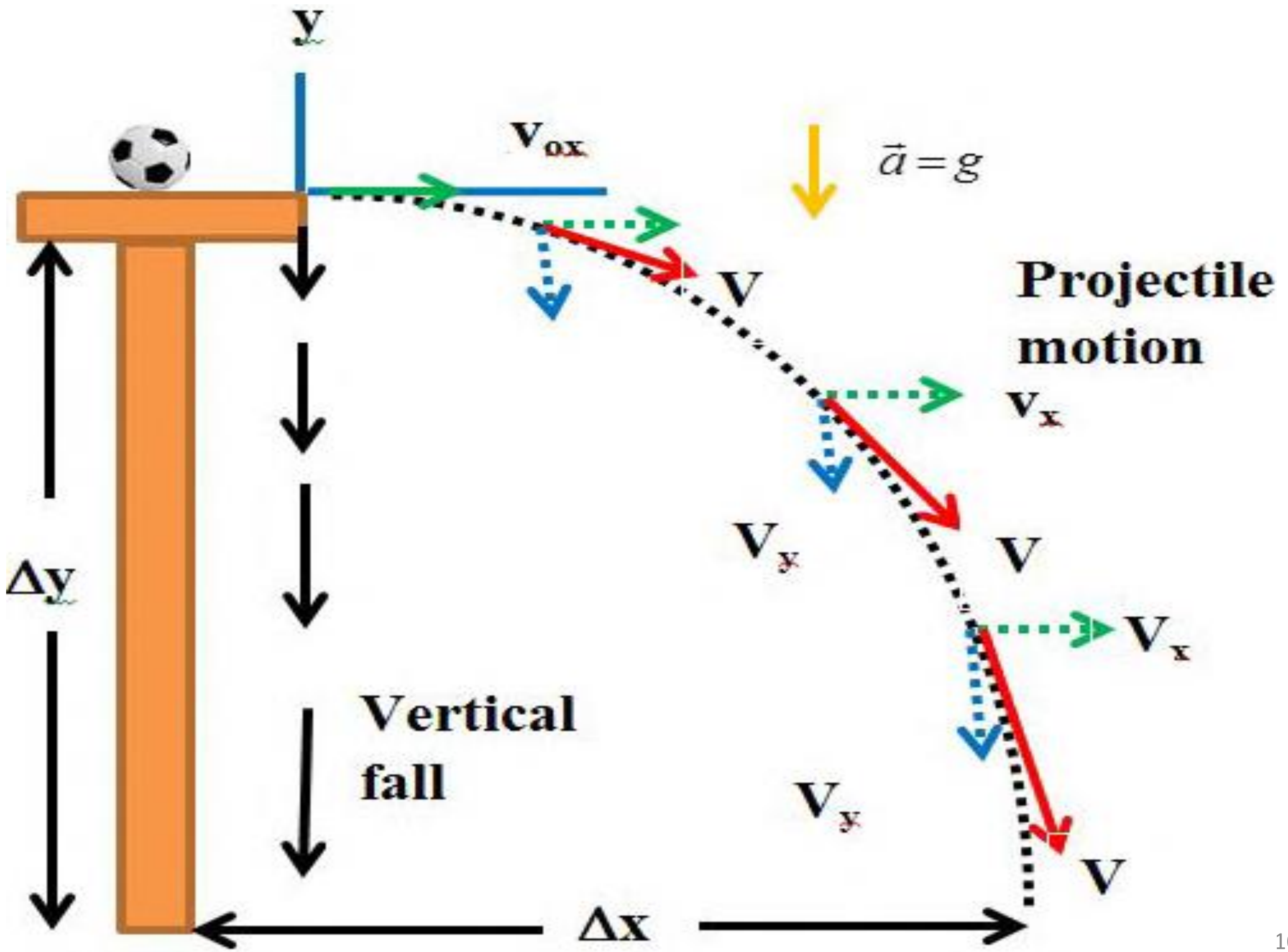
$$v_x = v_{0x}$$

- ✓ The horizontal distance traveled by the projectile at a time *t* is given by

$$\Delta x = v_{0x}\mathrm{t}$$

Vertical Motion:

- ✓ The vertical motion is influenced by gravity, causing the object to accelerate downward.


y
V_{ox}
$\vec{a} = g$
V
Projectile motion
V_x
V_y
V
V_x
Δy
Vertical fall
V_y
V
Δx

Vertical Motion Under Gravity

- ✓ vertical motion is subject to constant acceleration due to gravity.
- ✓ acceleration (g) is approximately -9.8 m/s² (downward).
- ✓ kinematic equations of motion for constant accelerated motion which are shown below.

$$v_y = v_{0y} + \mathrm{g}t$$

$$\Delta y(t) = v_{0y}t + \frac{1}{2}\mathrm{g}t^2$$

Where:

v_y = Final vertical velocity

v_{0y} = Initial vertical velocity

✓ The vertical initial velocity has no downward component,

$v_{0y} = 0$

$$v_y = \mathrm{g}t$$

$$\Delta y(t) = \frac{1}{2}\mathrm{g}t^2$$

Time of flight (t)

- is the duration it takes for the projectile to hit the ground.
- determined by;

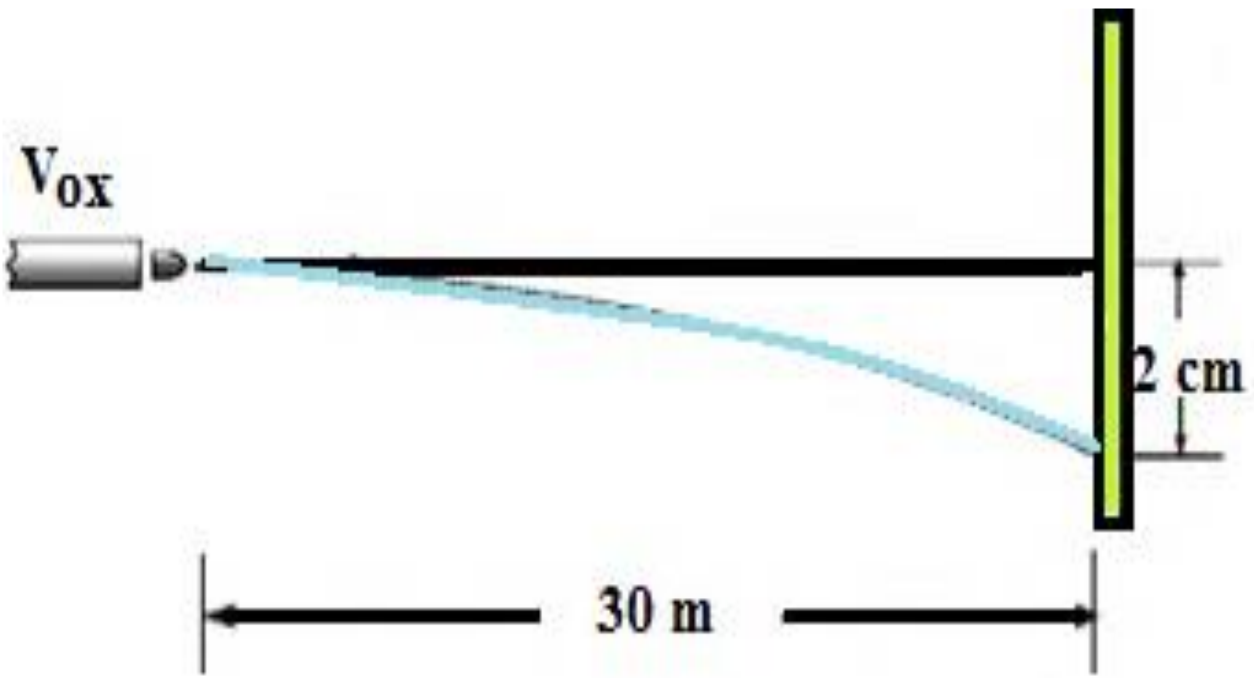
$$t = \sqrt{\frac{2\Delta y}{\mathrm{g}}}$$

Range (R): is the maximum horizontal distance travelled by a projectile.

$$R = v_{0x}\sqrt{\frac{2\Delta y}{\mathrm{g}}}$$

Examples

1. A rifle is aimed horizontally at a target 30m away as shown in Figure below. The bullet hits the target 2 cm below the aiming point.
   (a) What is the bullet's time of flight?
   (b) What is the initial velocity of the bullet?



2. A rescue airplane travelling at 360km/h(100m/s) horizontally dropped a food package from a height of 300m when it passes over a car driver stranded in a snowstorm.

(a) How long will it take the food package to reach the ground?

(b) How far from the car driver should the food package be dropped ?

ii) Inclined Projectile Motion

Definition: *Inclined projectile motion refers to the motion of an object that is projected at an angle to the horizontal plane.*

Initial Velocity: The object is projected with an initial velocity at an angle.

Horizontal and Vertical Components: The initial velocity is split into horizontal and vertical components.

Horizontal Motion: The horizontal velocity remains constant.

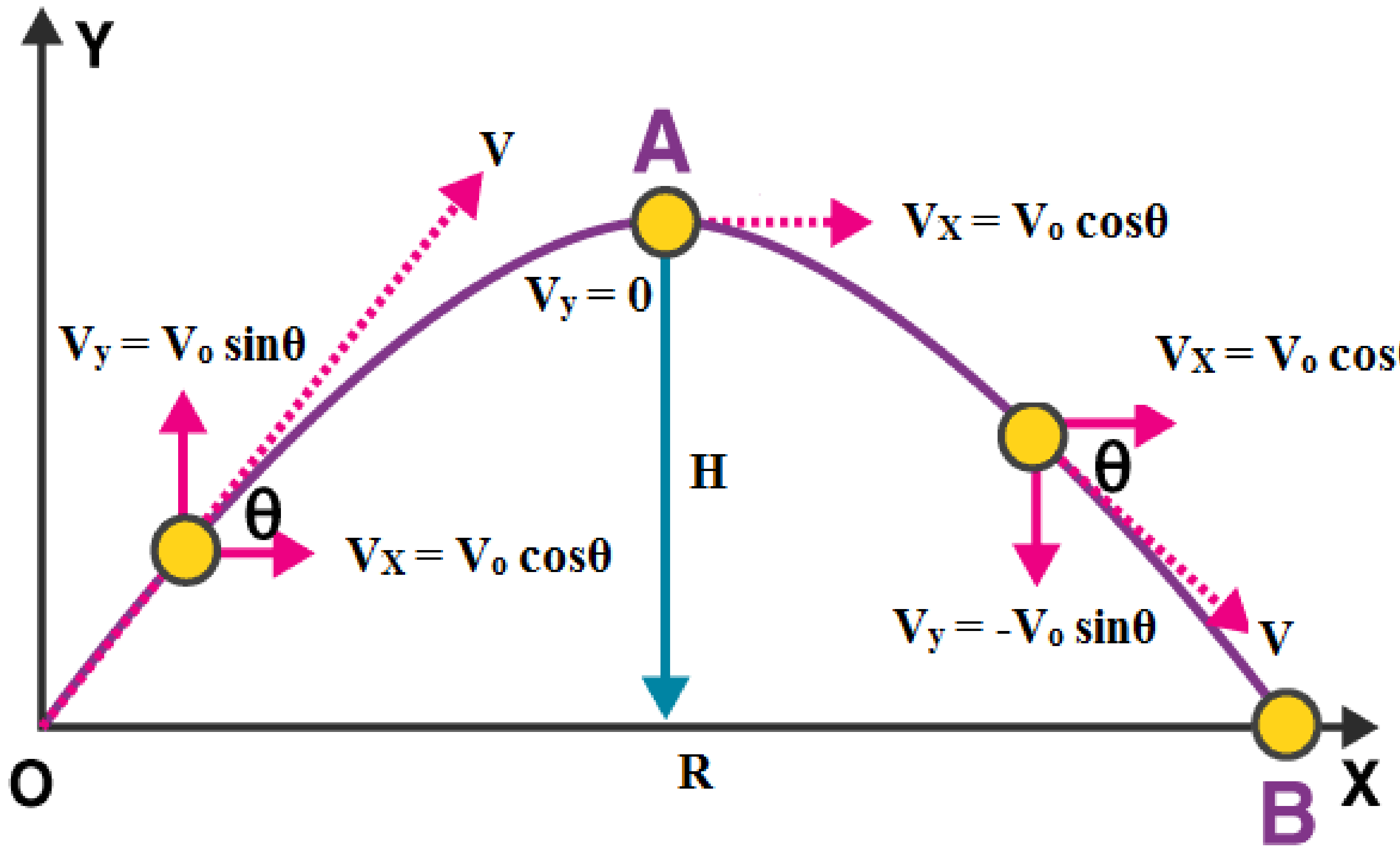
Vertical Motion: The vertical motion is affected by gravity, causing deceleration and acceleration.

Time of Flight: The total time in the air.

Maximum Height: The highest point reached by the projectile.

Range: The total horizontal distance covered.

Trajectory: The path is a parabolic curve.



In an inclined Projectile Motion:

- ✓ *the motion can be broken down into two components: horizontal and vertical.*
- ✓ *the two components operate independently of each other.*

Resolving the Initial Velocity

- *Horizontal component:* $v_x = v_0 cos\theta$
- *Vertical component:* $v_y = v_0 sin\theta$

Horizontal Component of Motion

- ✓ *the x-component of velocity* (v_x) *is constant throughout the flight.*
- ✓ *no force acts along the horizontal direction, hence no acceleration along the x-axis.*

Vertical Component of Motion

- ✓ the y-component of velocity (v_y) changes with time due to gravity.
- ✓ the acceleration is ($g = -9.8\ m/s^2$).
- ✓ at the maximum height, the vertical component of the velocity (v_y) becomes zero.
- ✓ after reaching maximum height, the projectile changes direction and starts to fall.

Equations of inclined projectile motion

- ✓ Horizontal velocity at any time t.

$$v_x = v_0\cos\theta \text{ (Constant)}$$

- ✓ Vertical velocity at any time t.

$$v_y = v_0\sin\theta + gt$$

Displacements of the projectile

There are two different types of displacements of the projectile motion:

- ✓ horizontal displacement at any time t.

$$\Delta x = v_o \cos\theta t$$

- ✓ vertical displacement at any time t:

$$\Delta y = v_o \sin\theta t + \frac{1}{2} g t^2$$

The time to reach maximum height is

$$t = \frac{v_o sin\theta}{g}$$

Time of flight

- ✓ It is the total time for which the projectile remains in flight.
- ✓ It depends on the initial velocity of the object and the angle of the projection, θ.

$$t_f = \frac{2v_0 sin\theta}{g}$$

this equation does not apply when the projectile lands at a different elevation than it was launched

Horizontal Range (R)

- ✓ is the total horizontal distance covered:

$$R = v_{0x} t_f = v_0 cos\theta \left(\frac{2v_0 sin\theta}{g}\right)$$

$$R = \frac{{v_0}^2 sin2\theta}{g}$$

Maximum vertical height (Δy_{max})

- ✓ occurs when the vertical component of velocity, v_y equals zero.
- ✓ once the projectile reaches its maximum height, it begins to accelerate downward.

$$0 = v_o \sin\theta t + \frac{1}{2} g t^2$$

- ✓ the time to cover the maximum height is: $t = \frac{v_o sin\theta}{g}$

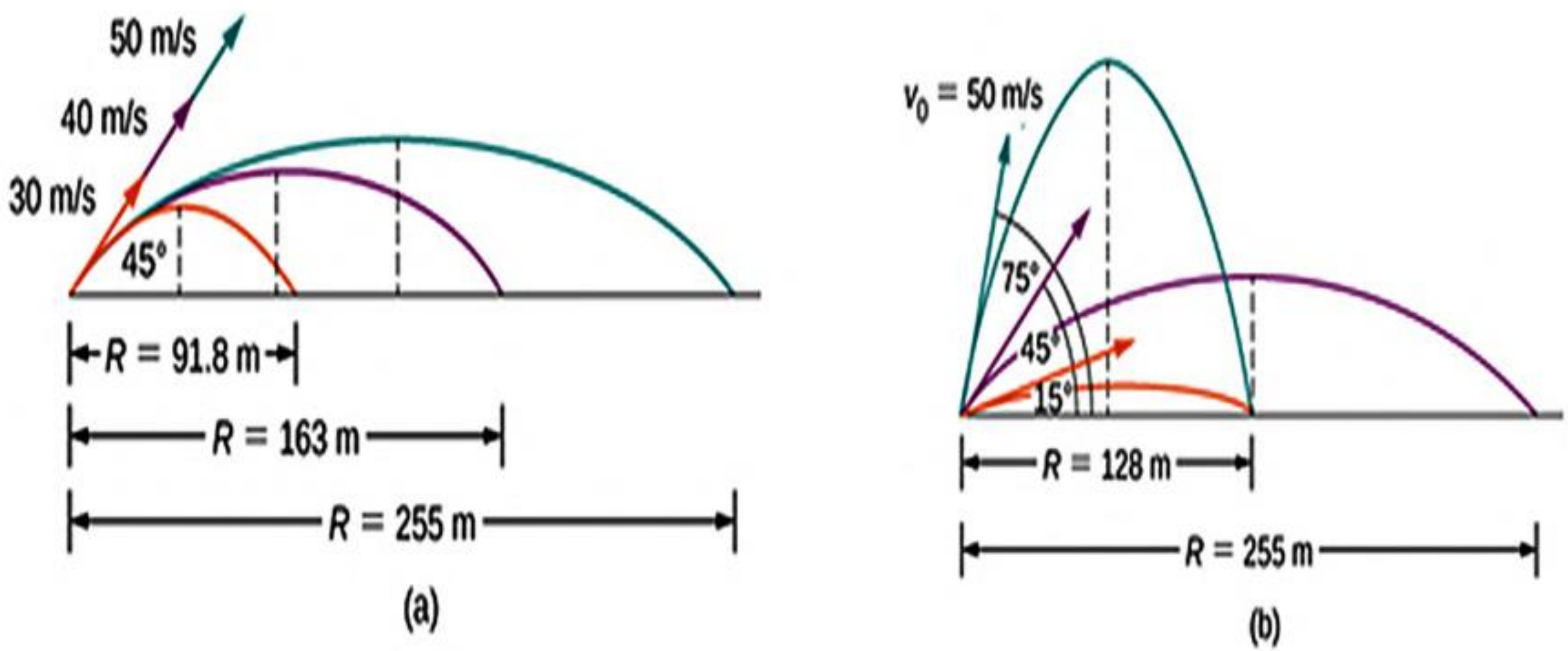
Then;

- ✓ the maximum height, $\Delta y_{max} = \text{H}$

$$\text{H} = \frac{v_0{}^2 sin^2\theta}{2g}$$

- The horizontal range is maximum at 45^0.
- It is interesting that the same range is found for two initial launch angles that sum to 90^0.
  - ✓ *The projectile launched with the smaller angle has a lower apex than the higher angle, but they both have the same range.*

introduction to projectile motion.mp4



Relation between Range and maximum height

Dividing the maximum height of the projectile by horizontal range.

$$\frac{H}{R} = \frac{\frac{v_0{}^2 sin^2\theta}{2g}}{\frac{v_0{}^2 sin2\theta}{g}} = \frac{\sin\theta sin\theta}{4sin\theta cos\theta} = \frac{tan\theta}{4}$$

$$H = \frac{R\tan\theta}{4}$$

Examples

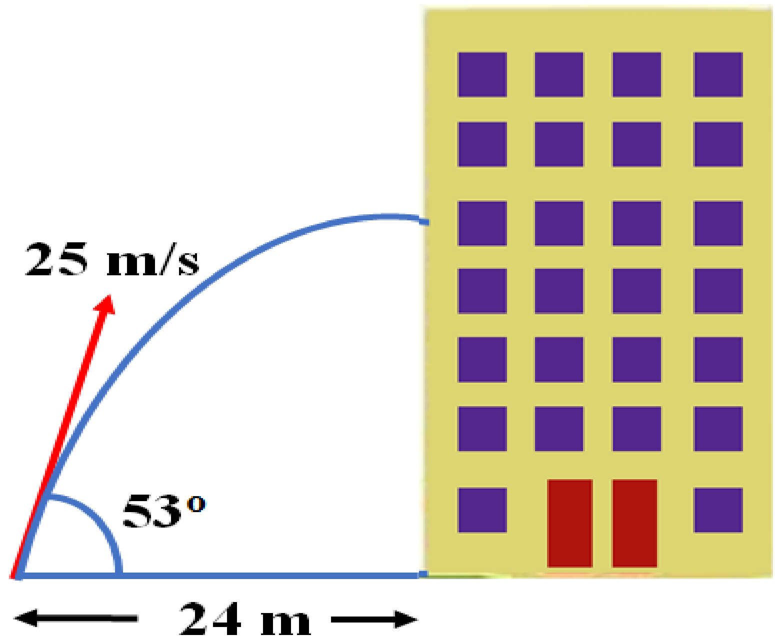
1. A football player kicked a ball at angle of 37^0 with the horizontal. The initial velocity of the ball is 40m/s.
   a) Find the maximum height reached and
   b) The horizontal range

2. You throw a ball with a speed of 25 m/s at an angle of 53^0 above the horizontal directly toward a wall, as shown in figure below. The wall is 24m from the release point of the ball.

(a) How long does the ball take to reach the wall?

(b) How far above the release point does the ball hit the wall?

(c) What are the horizontal and vertical components of its velocity as it hits the wall?



Applications of Projectile Motion: Understanding the Real-World Impact of Physics

1. **Sports Applications**

   Examples:

   - ✓ Basketball: calculating the arc of a shot.
   - ✓ Soccer: predicting the path of a kicked ball.
   - ✓ Golf: determining the trajectory and distance of a golf ball.

Importance: Athletes and coaches use these principles to improve performance.

2. Engineering Applications

Civil Engineering:

- ✓ Designing structures like bridges to withstand forces.
- ✓ Ensuring stability and safety.

Mechanical Engineering:

- ✓ Trajectory analysis in machines and vehicles.
- ✓ Applications in robotics and automated systems.

3. Military Applications

Ballistics:

- ✓ Calculating the trajectory of missiles and artillery shells.
- ✓ Enhancing accuracy and effectiveness in military operations.

Weapons Design:

- ✓ Designing firearms and other weapons systems.

4. Aerospace Applications

Satellite Launch:

- ✓ Predicting and controlling the trajectory of satellites.

Spacecraft Re-entry:

- ✓ Planning the re-entry paths to ensure safety and precision.

Missile Defense:

- ✓ Intercepting incoming projectiles.

5. Entertainment Applications

Video Games:

- ✓ Simulating realistic physics in shooting games.
- ✓ Designing gameplay mechanics involving projectiles.

Animation:

- ✓ Creating realistic motion in animated films.

6. Accident Reconstruction

Forensic Science:

- ✓ Reconstructing events in car accidents.
- ✓ Analyzing bullet trajectories at crime scenes.

Importance: *Assisting in legal cases and investigations.*

7. Meteorology Applications

Weather Prediction:

- ✓ Analyzing the motion of particles like raindrops or hail.

Debris Trajectory:

- ✓ Predicting the path of debris during storms or natural disasters.

2.2 Rotational Motion

- *is the motion of a body, in which all of its particles move in a circular motion with a common angular velocity about a fixed axis.*

rotational dynamics.mp4

For example;

- ✓ the rotation of Earth about its axis
- ✓ the rotation of the flywheel of a sewing machine
- ✓ rotation of ceiling fan
- ✓ rotation of wheels of a car, etc

➢ The rotation of an object about a fixed point can be of two directions: **clockwise** or **anticlockwise** direction.

- Rigid body is a body with perfectly defined and unchanging shape that is no matter how the body moves , the distance between any two particles within the body remains constant.

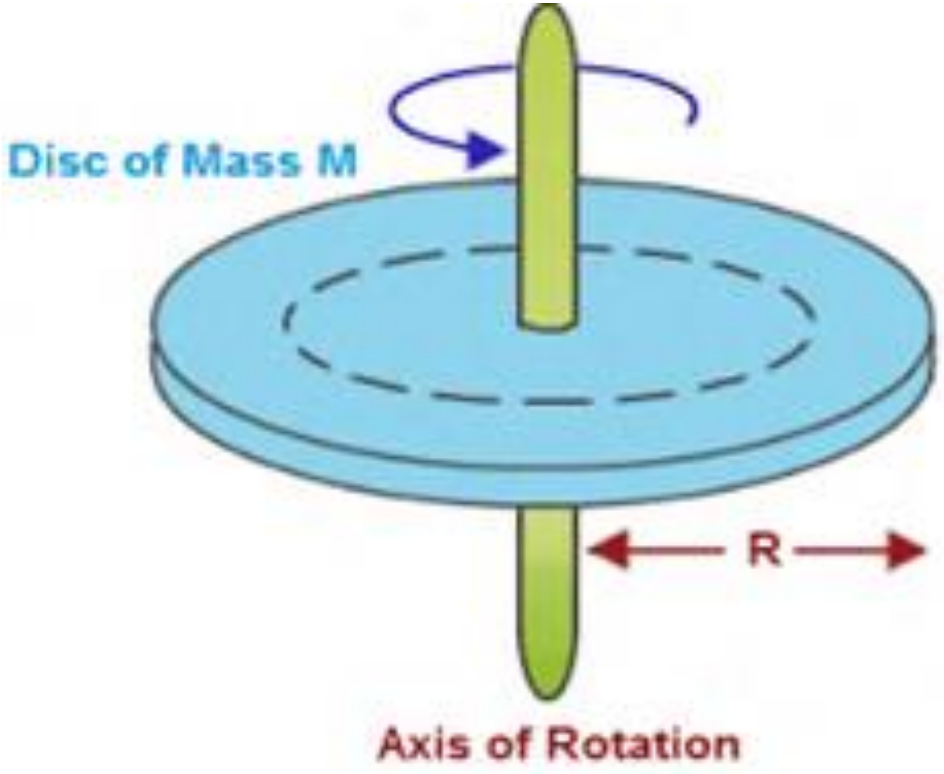


Figure Rotation of a disc of mass M about a fixed axis.

Angular displacement and angular velocity

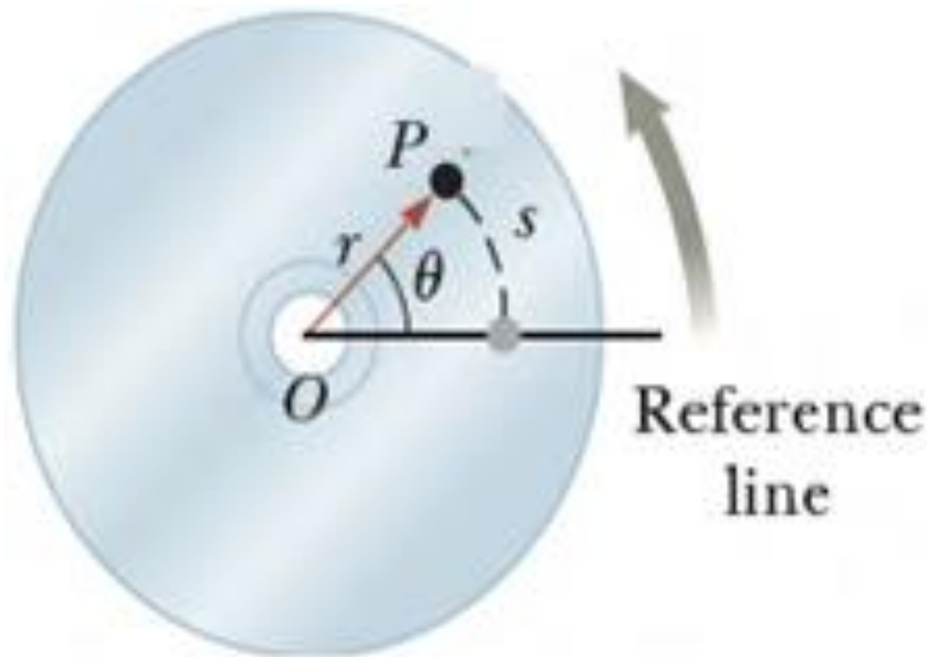
Angular Displacement($\Delta\theta$)

Consider the disc rotates about a fixed axis perpendicular to the plane, a small element of the disc modeled as a particle at P is kept at a fixed distance r from the origin and rotates about O in a circle of radius r shown below.

- The arc length *s* is related to the angle θ through the relationship:

$$s = r\theta$$

  - ✓ *every other particle on the object rotates through the same angle*



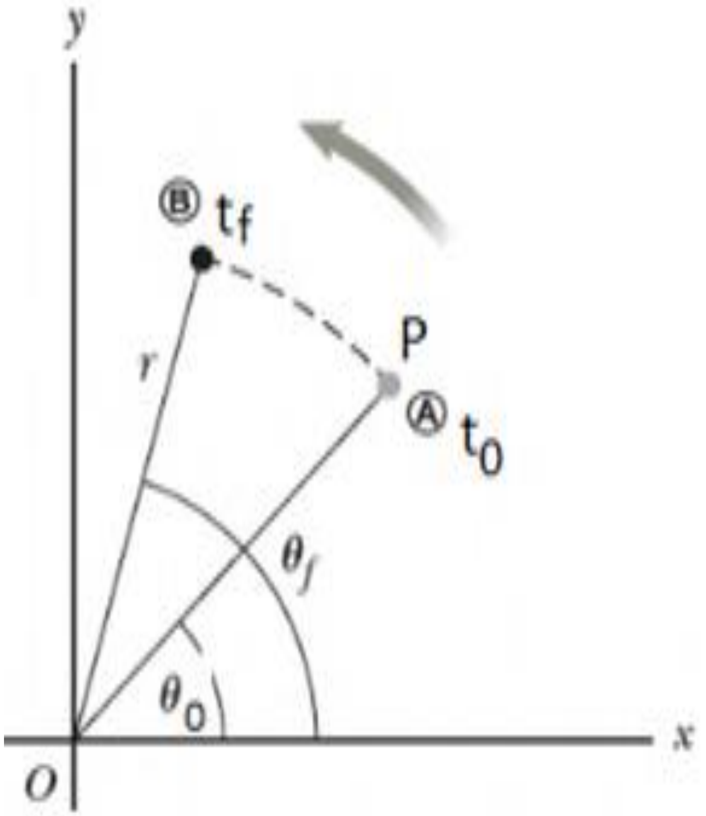
As the particle in our disc travels from position A to position B in a time interval Δt as in Figure below, the reference line fixed to the object sweeps out an angle $\Delta\theta$.

- ✓ *the angular displacement ($\Delta\theta$) of the rigid object is given by;*

$$\Delta\theta = \theta_f - \theta_0$$

- ✓ *physicists use radians or degrees to measure angular displacement.*
- ✓ *A radian is convenient because it naturally expresses angles in terms of π, since one complete turn of a circle (360 degrees) equals 2π radians.*

$$1rev = 360\ degrees = 2\pi\ rad.$$



Angular Velocity(ω)

- ✓ **is** a measure of how fast an object or
- ✓ is the rate at which angular displacement occurs can vary.

$$\omega_{av} = \frac{\theta_f - \theta_0}{t_f - t_0} = \frac{\Delta\theta}{\Delta t}$$

Angular velocity has units of radians per second (rad/s).

Angular Acceleration (α)

Angular acceleration of a rotating rigid object is defined as the ratio of the change in the angular speed to the time interval Δt during which the change in the angular speed occurs:

$$\alpha_{av} = \frac{\omega_f - \omega_0}{t_f - t_0} = \frac{\Delta\omega}{\Delta t}$$

Angular acceleration has units of radians per second squared (rad/s^2).

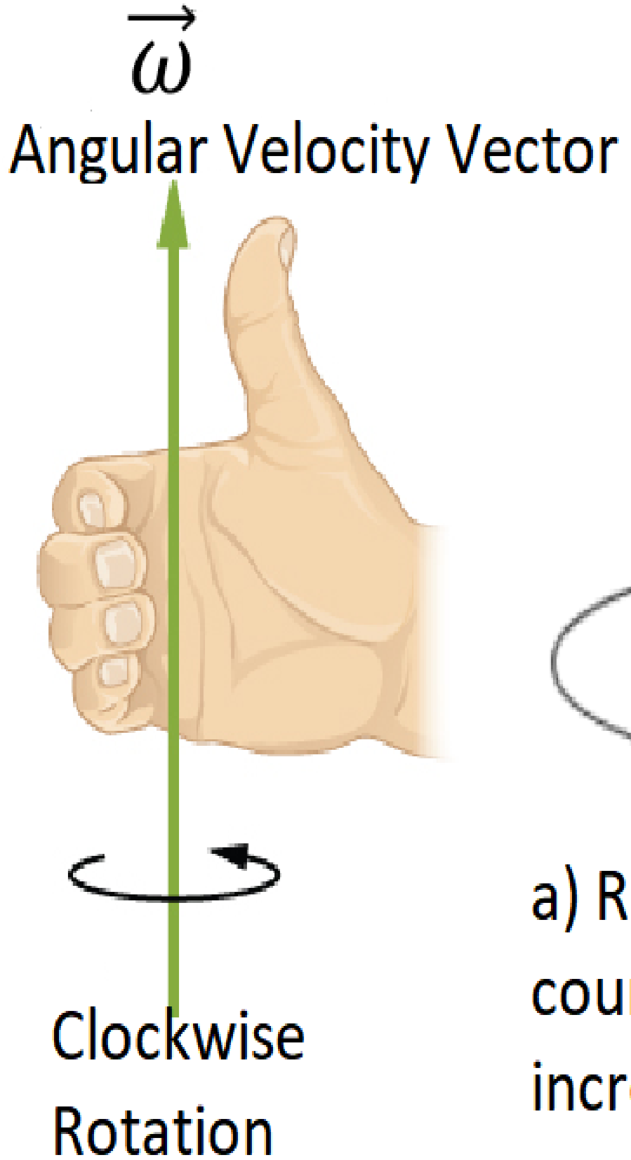
Direction of angular velocity and angular acceleration

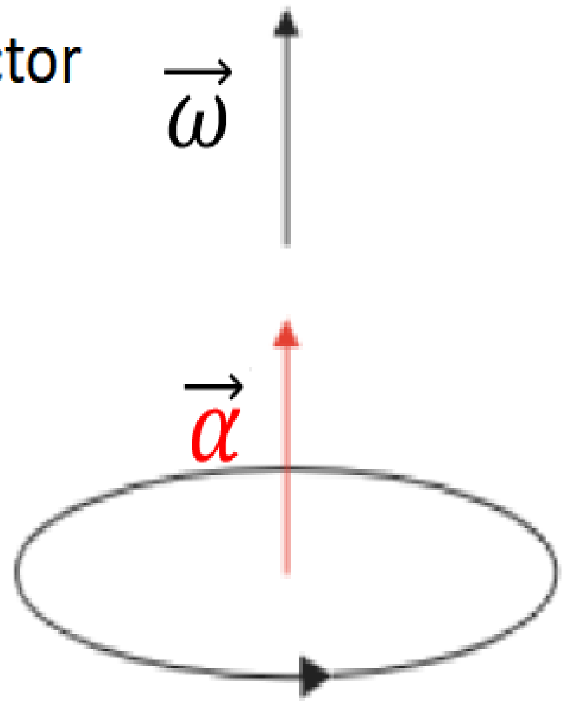
- For rotation about a fixed axis, the direction that uniquely specifies the rotational motion is the direction along the axis of rotation.
  - ✓ *the directions of ω and α are along this axis* ⇨ demon-strated by using *right-hand rule*

When the four fingers of the right hand are wrapped in the direction of rotation, the extended right thumb points in the direction of ω.

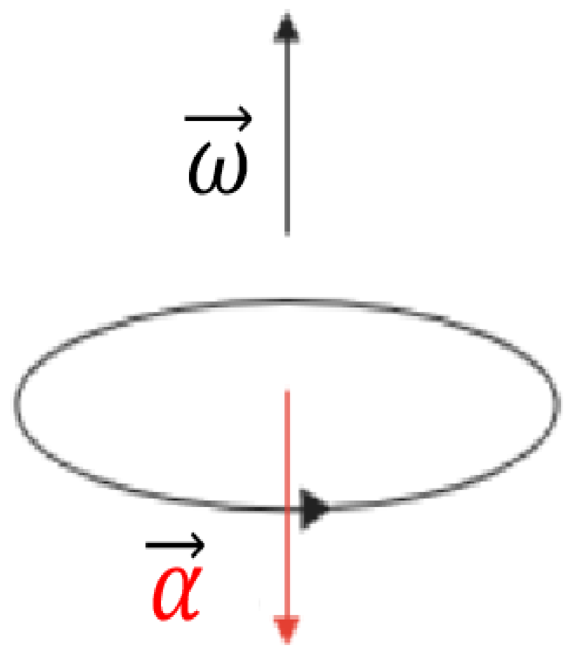
right hand rule.mp4

- ✓ *The direction of α (follows from its definition) is in the same direction as ω if the angular speed is increasing in time, and it is antiparallel to ω if the angular speed is decreasing in time.*





a) Rotation rate counterclockwise and increasing



b) Rotation rate counterclockwise and decreasing

Equations of Motion for Uniform angular acceleration

A set of kinematic equations exist for rotational motion just as they do for translational motion.

- ✓ For constant angular acceleration;

$$\alpha_{av} = \frac{\omega_f - \omega_0}{t_f - t_0} = \frac{\Delta\omega}{\Delta t} = \text{constant}$$

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\omega_0 + \omega_f}{2}$$

- ✓ The five important equations of uniform constant angular acceleration are:

$$1.\ \omega_f = \omega_0 + \alpha\Delta t$$

$$2.\ \Delta\theta = \left(\frac{\omega_0 + \omega_f}{2}\right)\text{t}$$

$$3.\ \theta = \omega_0\Delta t + \frac{1}{2}\alpha\Delta t^2$$

$$4.\ \Delta\theta = \omega_f \Delta t - \frac{1}{2}\alpha\Delta t^2$$

$$5.\ {\omega_f}^2 = {\omega_0}^2 + 2\alpha\Delta\theta$$

- *The kinematics for rotational motion is completely analogous to linear (or translational) kinematics.*

Activity! Write the five linear kinematics equations which are analogous to above equations.

Examples

1. What is the average angular velocity of a rotating wheel if its angular speed changes from 30 rad/s to 50rad/s in 2 seconds?

2. A rotating wheel has an initial angular velocity of 10rad/s and accelerates at 2.5rad/s^2.
   (a) How many revolutions are completed in 30 second?
   (b) What is angular speed of the wheel at $t = 20$ s?

3. A car's wheel has an initial angular velocity of 6 rad/s and a constant angular acceleration of 3rad/s^2. Calculate the angular velocity after 100rev?
4. A wheel has a radius of 20 cm and accelerates from rest to 15rev/s in 30 seconds. What is the magnitude of the tangential acceleration of a point at the tip of the wheel?
5. A car accelerates from 20m/s to 24m/s in 5sec. Calculate the angular acceleration of the wheels of the car if the radius of the wheels is 40cm.
6. A boy rides a bicycle for 5minutes. The wheel with radius of 30cm completes 2000 rev during this time. Calculate.
   (a) the average angular velocity of the wheel and
   (b) the linear distance traveled by the bicycle in 5min

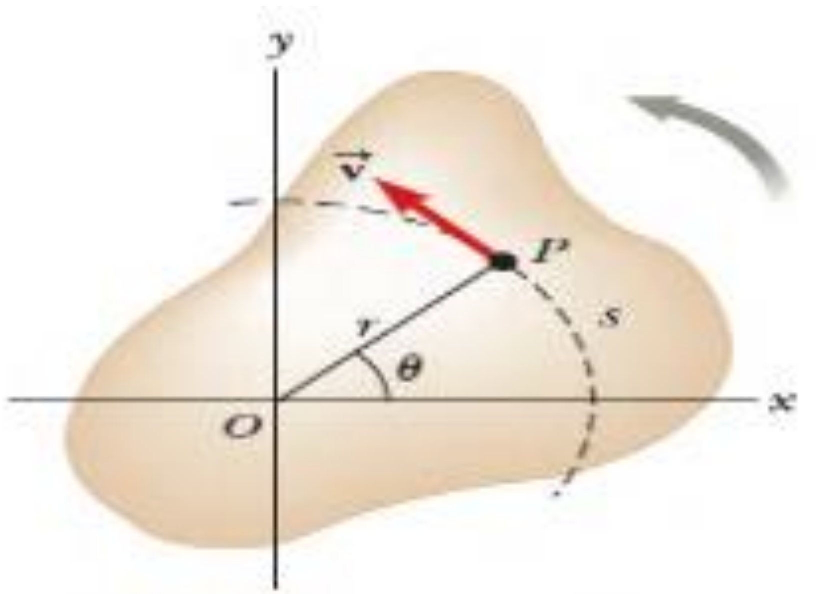
Activity!

A fly wheel is rotating with an angular velocity of 2 rad/s acted by an acceleration of $\frac{1}{\pi}$ rad/s^2. How long will it take to complete three revolutions?

Relationship between Angular and Translational Quantities

Consider a **rigid object rotates about a fixed axis** as in Figure below, every particle of the object moves in a circle whose center is on the axis of rotation.

rolling motion.mp4



- Because point P, in the Figure moves in a circle, the translational velocity vector $\vec{v}$ is always tangent to the circular path and hence is called tangential velocity.

- The magnitude of the tangential velocity of the point P is given by:

$$v = \frac{\Delta s}{\Delta t} \text{ but } \Delta s = r\Delta\theta$$

$$v = \frac{r\Delta\theta}{\Delta t}, \omega = \frac{\Delta\theta}{\Delta t}$$

tangential velocity.mp4 direction of angular velocity.mp4

$$\boldsymbol{v = r\omega}$$

Note

- Therefore, although every point on the rigid object has the same angular speed, not every point has the same tangential speed because r is not the same for all points on the object.
  - ✓ *The tangential speed of a point on the rotating object increases as one moves outward from the center of rotation.*

The magnitude of angular acceleration of the rotating rigid object related to the tangential acceleration of the point P as follows;

$$a = r\alpha \qquad \text{(show!)}$$

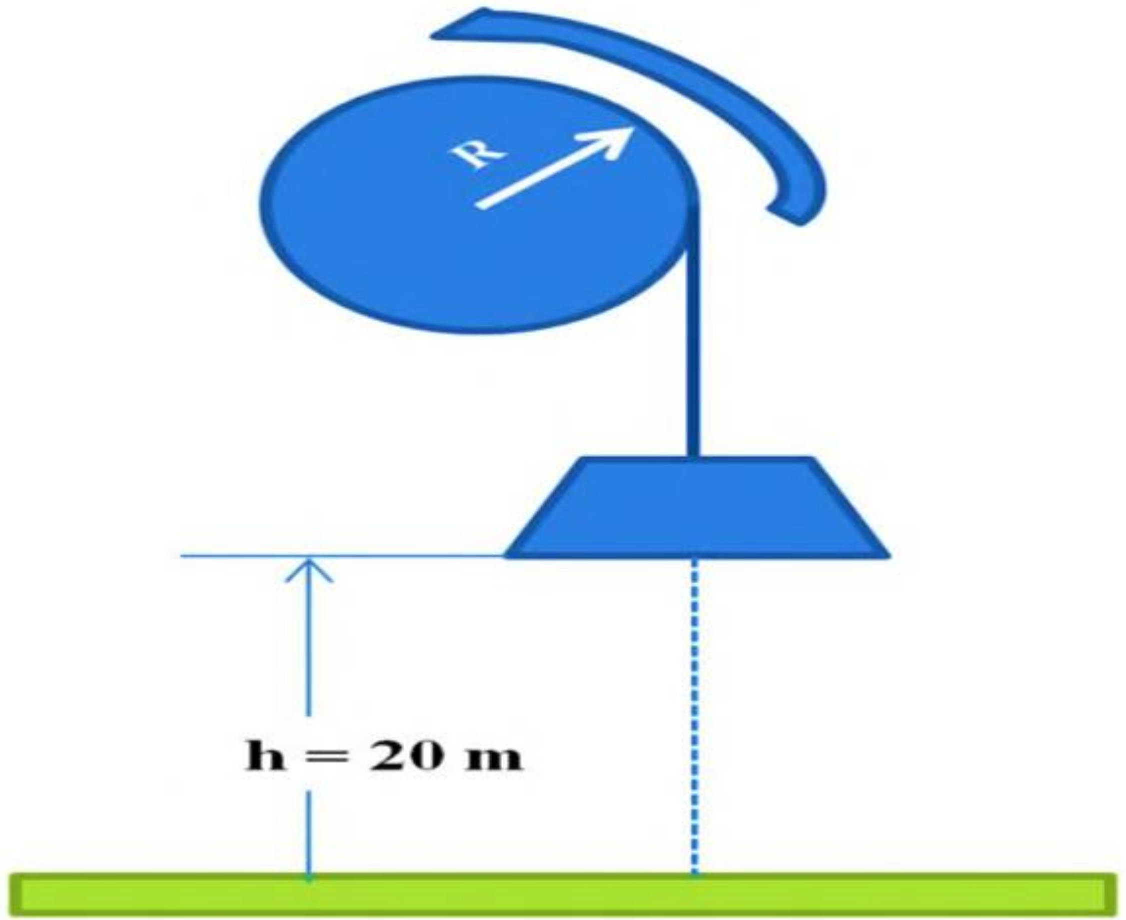
Note

- The tangential component of the translational acceleration of a point on a rotating rigid object equals the radius multiplied by the angular acceleration.

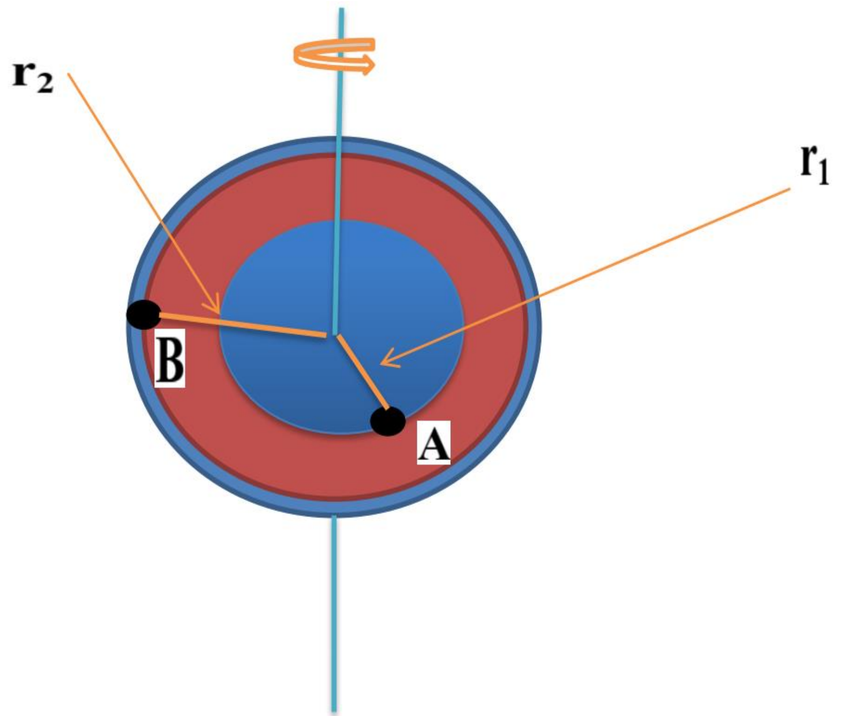
Examples

1. The angular velocity of a bicycle wheel is 18 rad/s. If the radius of the wheel is 40cm, what is the speed of the bicycle in m/s?
2. A rope is wrapped many times around a pulley of radius 20 cm. What is the average angular velocity of the pulley if it lifts a bucket to 10 m in 5 s?

3. A rope is wrapped many times around a pulley of radius 50 cm as shown in figure below. How many revolutions of the pulley are required to raise a bucket to a height of 20 m?



4. Consider flat rotating disk to angular velocity of 20 rad/s starting from rest in 4 seconds, as shown in figure below
   (a) What is the average angular and linear acceleration for particle A (r_1= 20 cm)?
   (b) What is the average angular and linear acceleration for particle B (r_2= 40 cm)?



Activity! (connect with circular motion!)

A particle moves in a circle of radius 5 cm with constant speed and time period 0.2πs. calculate the acceleration of the particle.

2.3 Rotational Dynamics

- ✓ studies the motion of object rotating around an axis, focusing on forces (torques) that cause rotation.

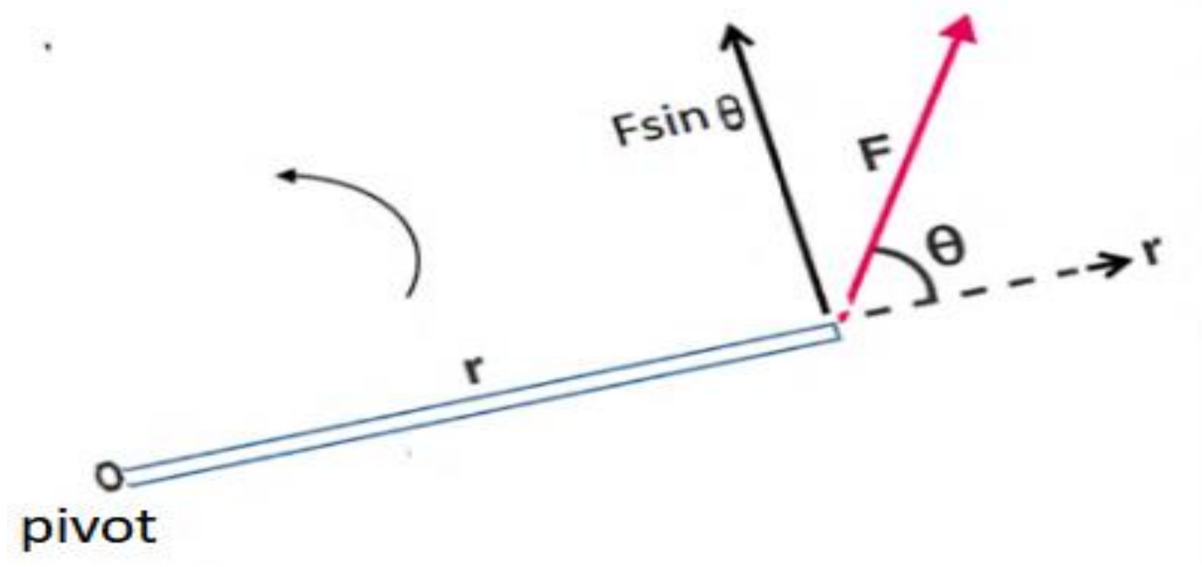
Torque (τ)

- ✓ is the rotational effect of force.
- ✓ is what cause an object to acquire angular acceleration.

Let F be a force acting on an object and r be the distance from the axis of rotation to the point of application of the force as shown in Fig. below. The magnitude of the torque is given by

$$\tau = rFsin\theta$$

where θ is the angle between r and F when they are drawn from the same origin.

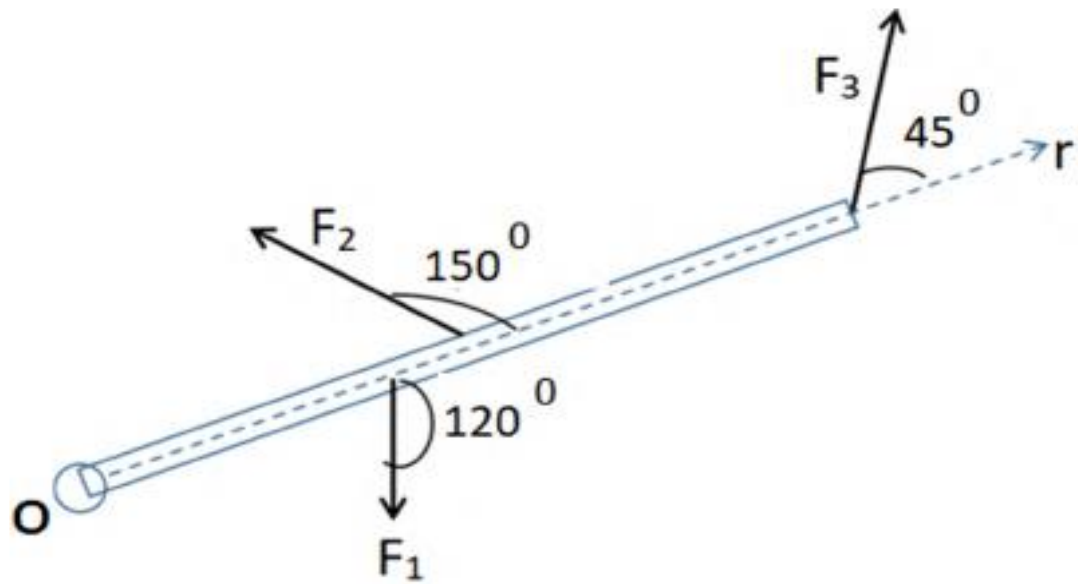


Torque - is a vector quantity.

- its SI unit is N.m.
- is along the axis of rotation which is determined by a right hand rule. That is curl the fingers of your right hand in the direction of the rotation your thumb points the direction of the torque.

Example

The body in below is pivoted at O. Three forces act on it in the directions shown: F_1 = 10 N at 8.0m from O; F_2 = 12 N at 10 m from O; and F_3 = 8 N at 20 m from O. What is the net torque about O?



Moment of Inertia (I) of a body

- is the quantitative measure of rotational inertia.

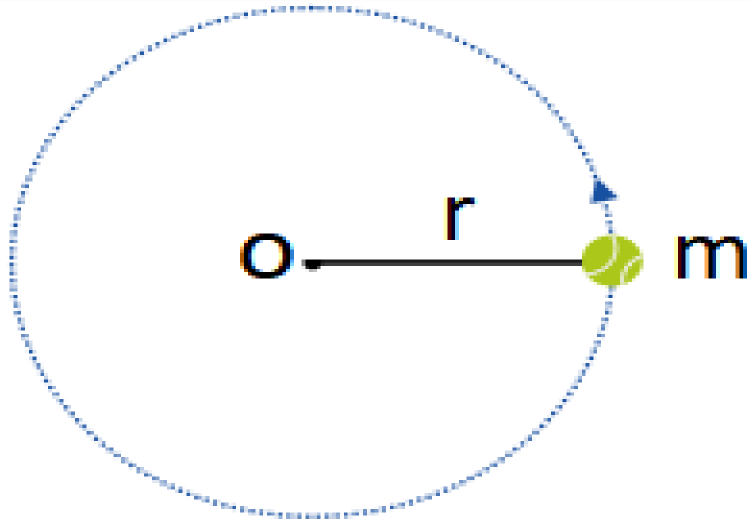
Question! What is the translational equivalent of moment of inertia of a body?

- The greater the moment of inertia of a rigid body or system of particles, the greater is its resistance to change in angular velocity about a fixed axis of rotation.
- The moment of inertia depends on the mass, shape of a body and axis of rotation of the body.
- For a single point mass rotating at radius r from the axis of rotation the moment of inertia is;

$$I = mr^2$$

Moment of inertia – SI unit $kg.m^2$

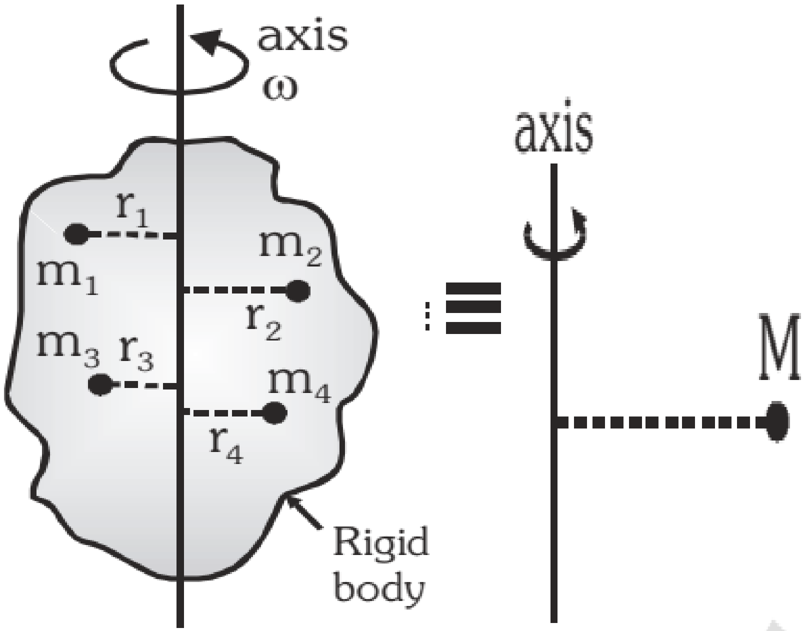
- is a scalar quantity.



- The moment of inertia for more than one particle about a fixed axis is:

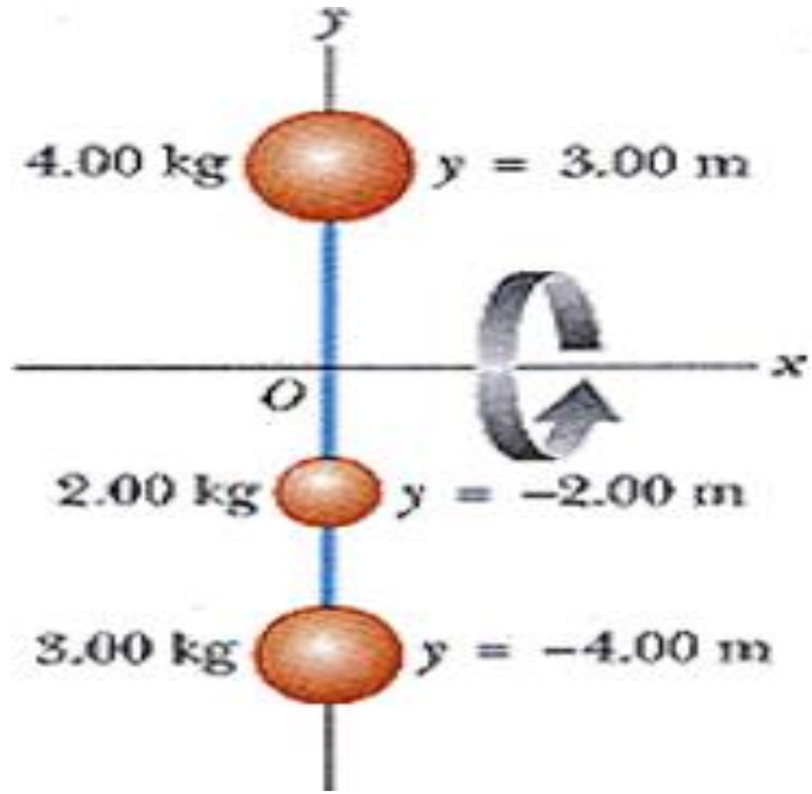
$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 \ldots$$

$$= \sum mr^2$$



Example

Three particles are connected by rigid rods of negligible mass lying along the y-axis as shown in Figure below the system rotates about the x-axis with angular speed of 2 rad/s, find the moment of inertia about the x-axis.



Activity!

Three point masses, each of mass m, are placed at the corners of an equilateral triangle of side L. Find the moment of inertia of the system about an axis passing through one of the corners perpendicular to the plane of the triangle.

Note!

Moment of inertia does not depend on :-

- ✓ angular velocity
- ✓ angular acceleration
- ✓ torque
- ✓ angular momentum

Torque and angular acceleration

- When a number of individual forces act on a rotating object, we can compute the net torque:

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \cdots$$

- The net torque to angular acceleration α, by analogy with Newton's second law of motion (F = ma), we replace m by I and a by α.

$$\tau = I\alpha$$

- The angular acceleration of a rotating object is proportional to the net torque on the object.

Example

1. When a torque of 36Nm is applied to a certain wheel, the wheel acquires an angular acceleration of 24 rad/s^2. Find the rotational inertia of the wheel.

2. A motor capable of producing a constant torque 100Nm and a maximum rotation speed of 150rad/s is connected to a flywheel with rotational inertia 0.1kg.m^2.
   (a) What angular acceleration will the flywheel experience as the motor is switched on?
   (b) How long will the flywheel take to reach the maximum speed if starting from rest?
3. A disc with moment of inertia 2 kg.m^2 changes its angular speed from 3rad/s to 8rad/s by a net torque of 50Nm.How long will the disc take to change its angular speed?

Activity!

Two objects solid sphere & hollow sphere have the same mass. If same torque is applied to both, which object will have a greater angular acceleration? Explain!

2.4 Planetary motion and Kepler's Laws

The study of the planetary motion: the major steps involved were :

- The hypothesis about planetary motion given by Nicolaus Copernicus (1473–1543).

↓

- The careful experimental measurements of the positions of the planets and the Sun by Tycho Brahe (1546–1601).

↓

- Analysis of the data and the formulation of empirical laws by Johannes Kepler (1571–1630).

↓

- The development of a general theory by Isaac Newton (1642–1727).

Kepler's Laws

Keplers law.mp4

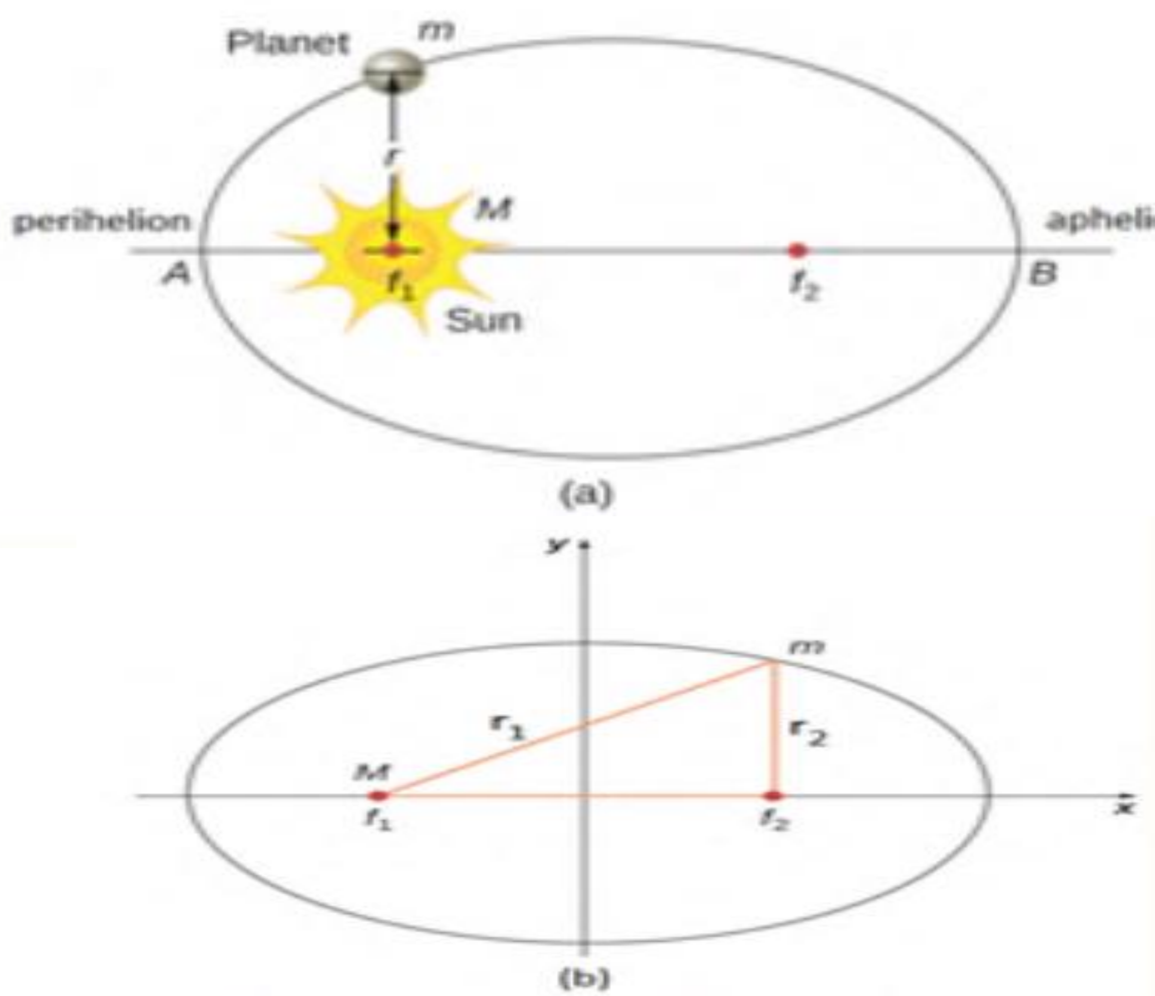
i) Kepler's First Law (Law of Ellipses)

Statement: Planets move in elliptical orbits with the Sun at one focus.

Ellipse: A geometric shape that is elongated. It has two foci, and the Sun is located at one of these foci.

Difference from Circle: A circle is a special case of an ellipse where both foci are at the same point (the center).

- *An ellipse is a closed curve such that the sum of the distances from a point on the curve* $(r_1 + r_2)$ *to the two foci,* f_1 *and* f_2 *is constant.*



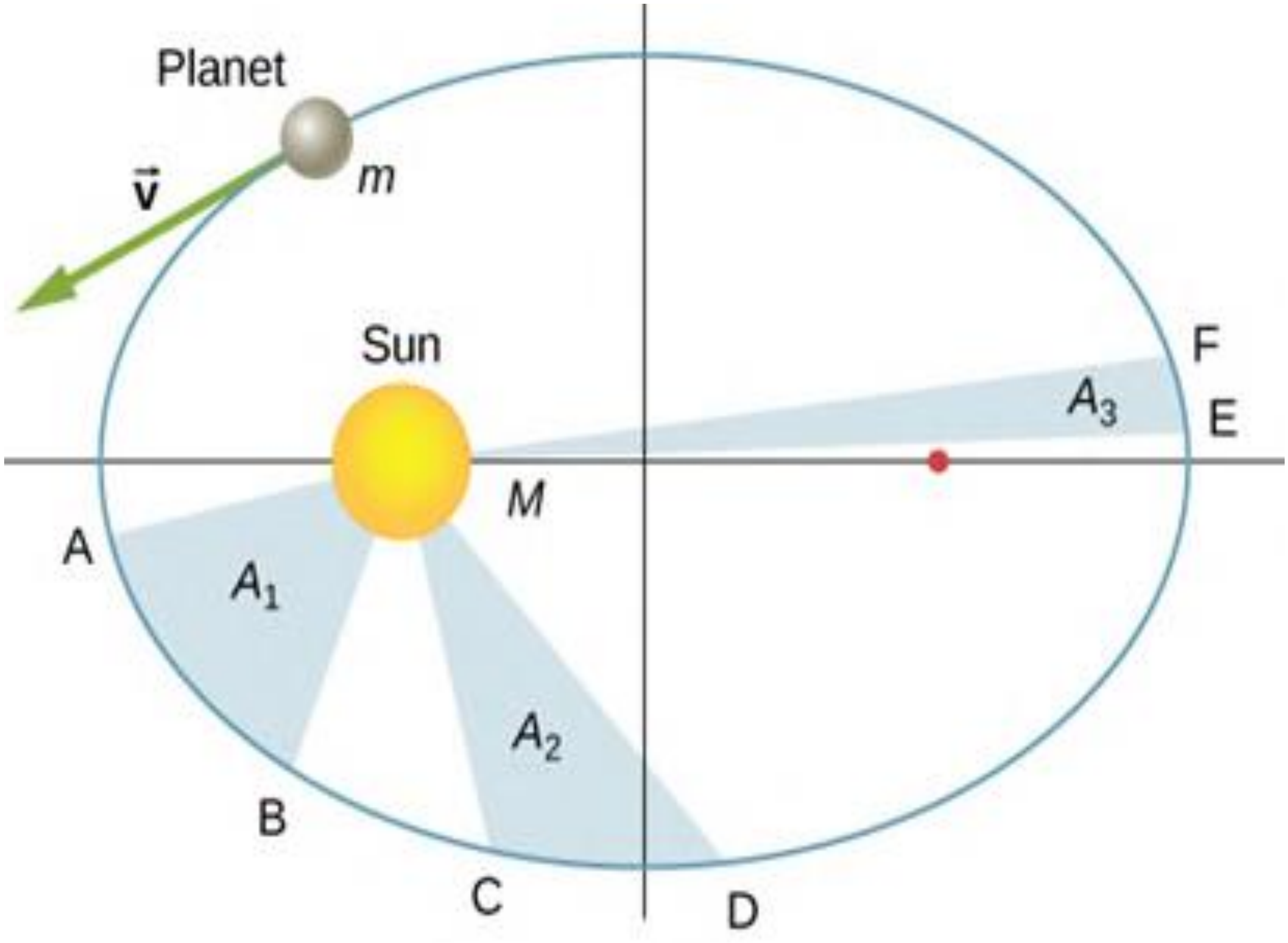
https://phet.colorado.edu/sims/html/keplers-laws/latest/keplers-laws_all.html

ii) Kepler's Second Law (Law of Equal Areas)

Statement: A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.

Implication: A planet moves faster when it is closer to the Sun and slower when it is farther from the Sun, ensuring that the area swept by the line segment in any given time period is constant.

- The time it takes a planet to move from position A to B, sweeping out area A_1,is exactly the time taken to move from position C to D, sweeping area A_2 and to move from E to F, sweeping out area A_3 These areas are the same: $A_1 = A_2 = A_3$


Planet
$\vec{v}$
m
Sun
M
F
E
A
B
C
D
A_1
A_2
A_3

iii) Kepler's Third Law (Law of Harmonies)

Statement: The square of the orbital period (T^2) of a planet is directly proportional to the cube of the semi-major axis (a^3) of its orbit.

Formula: $\frac{T^2}{a^3}$ = constant (K)

$$\frac{T^2{}_1}{a^3{}_1} = \frac{T^2{}_2}{a^3{}_2}$$ (for two planets)

Implication: *This law relates the time it takes for a planet to complete its orbit to its average distance from the Sun.*

Note

- Kepler's third law equation is valid for both circular and elliptical orbits.

- K- is a proportionality constant which is nearly the same for all planets.
  - is independent of the mass of the planet.

The orbital period and average distance from sun (orbital radius) for Earth and mars as given in the table below.

	Period(s)	AverageDistance (m)	$T^2/R^3 (s^2/m^3)$
Earth	3.156×10^7 s	1.4957×10^{11}	2.977×10^{-19}
Mars	5.93×10^7 s	2.278×10^{11}	2.975×10^{-19}

- $\frac{T^2}{a^3}$ for both the Earth and Mars is the same.

Kepler's Laws.mp4

Examples

1. Earth has an orbital period of 365 days and its mean distance from the Sun is 1.495×10^8 km. The planet Pluto's mean distance from the Sun is 5.896×10^9 km. Using Kepler's third law, calculate Pluto's orbital period in Earth days?
2. If Saturn is on average 9 times farther from the sun than the earth is, how long is its year in terms of earth year.

Activity!

What are the physical interpretation of keplers first and second law?

Physical interpretation of kepler's 1st law:

- ✓ planets do not orbit the sun in perfect circles, but rather in elliptical paths.
- ✓ the sun is not at the center of these ellipses, but at one of the focal points.

Physical interpretation of kepeler's 2nd law:

- ✓ changing speed of a planet as it orbits the sun.
- ✓ reflects conservation of angular momentum.

Discussion Questions

1. **What is an orbit?**
   - ✓ The path a celestial body follows as it revolves around another body, such as a planet around the Sun.
2. **What is the shape of an orbit?**
   - ✓ An ellipse, with the Sun at one of the foci.
3. **What's in the middle of the orbit?**
   - ✓ The Sun, located at one of the two foci of the elliptical orbit.
4. **What is the difference between a circle and an ellipse?**
   - ✓ A circle has a constant radius from its center, while an ellipse has two foci and varying distances from these points.

Applications of Kepler's Laws

Problem-Solving Equations:

1. Orbital Period Calculation:

- ✓ Use Kepler's Third Law to find the orbital period of a planet if the semi-major axis is known.

  Example: If the semi-major axis of a planet's orbit is 1AU, its period is 1 year.

2. Area Swept Calculation:

- ✓ Apply Kepler's Second Law to determine the changing speed of a planet as it moves along its elliptical orbit.

  Example: Compute the area swept by the planet in a specific time frame to understand its varying speed.

2.5 Newton's law of Universal Gravitation

Brainstorming Question

Question: Suppose the Sun's gravity is suddenly switched off, what will happen to the planets?

Discussion Prompt: Think about how the absence of the Sun's gravitational pull would affect the planets' orbits.

Newton's Insight

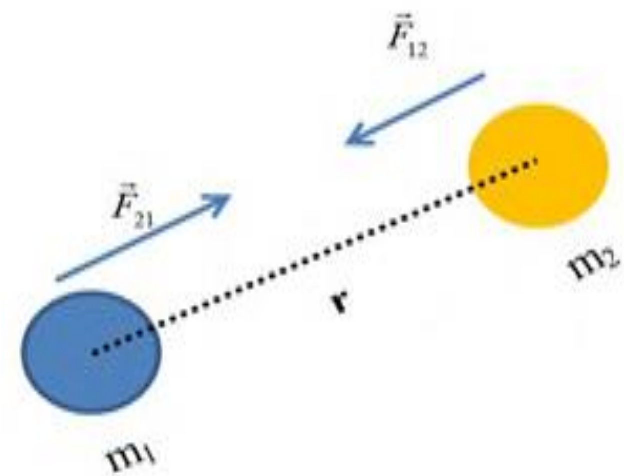
Galileo's Observation: Heavy and light objects fall at the same rate in a vacuum.

Newton's Contribution: Realized that the force causing an apple to fall is the same force keeping the Moon in orbit.

Newton's Universal Law of Gravitation

Statement: *Every particle of matter attracts every other particle with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers.*

Formula: $F = G\frac{m_1 m_2}{r^2}$



Where;

F: Gravitational force

G: Universal gravitational constant

(= $6.67 \times 10^{-11} Nm^2/kg^2$

determined by Hennery Cavendish)

m_1 & m_2: Masses of the two objects

r: Distance between the centers of the two masses

Gravitational Force Characteristics

- **Always Attractive:**
  - ✓ Gravity always pulls objects towards each other.
- **Depends on Masses and Distance:**
  - ✓ The force depends on the masses involved and the distance between them.
- **Direction:**
  - ✓ Force is directed along the line joining the centers of mass of the two bodies.

Understanding Universal law of Gravitation!.mp4

How Newton's Law Extends Kepler's Laws?

Kepler's Laws: Describe planetary motion in terms of elliptical orbits and periods.

Newton's Contribution: Provides a theoretical basis for Kepler's empirical laws through gravitational forces.

Examples

1. A 10 kg mass and a 100 kg mass are 1 meter apart. What is the force of attraction between them?
2. If a person has a mass of 60.0 kg, what would be the force of gravitational attraction on him at Earth's surface?

Discussion Question

Question: What keeps the planets in orbit? Explain.

Prompt: Consider the role of gravitational force and centripetal force in maintaining planetary orbits.

Summary

Gravitational Force: Determined by masses and distance; always attractive.

Newton's Law: Provides the foundation for understanding gravitational interactions in the universe.

Kepler's Laws: Extended and explained by Newton's gravitational theory.

Discussion points

1. By what factor would a person's weight at the surface of Earth change if Earth had its present mass but eight times its present volume?
2. By what factor would a person's weight at the surface of Earth change if Earth had its present size but only one-third its present mass?

Exercise!

Combining Newton's 2nd law and Newton's universal gravitational law, determine acceleration due to gravity of the Earth.

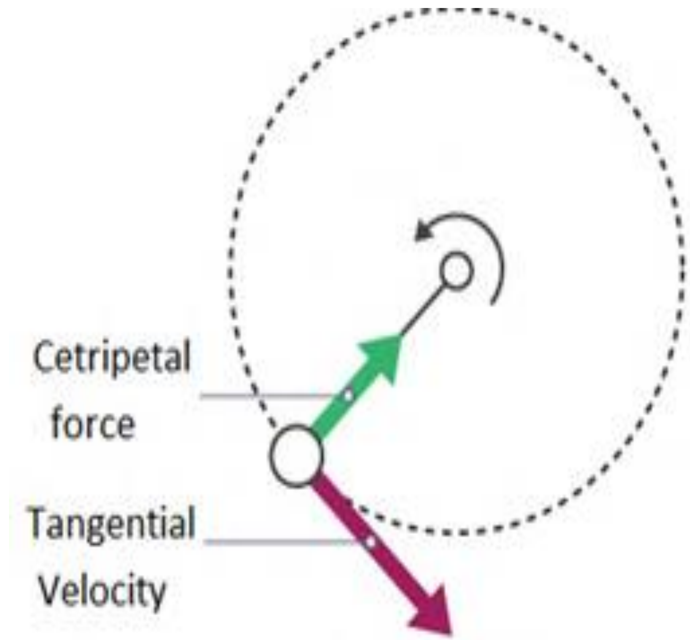
Derivation of Kepler's third law from Newton's law of universal gravitation.

The source for the centripetal force in the solar system is the gravitational force of the sun.

$$F_c = \frac{mv^2}{r} = F_G$$

$$\frac{m_p v^2}{r} = \frac{GM_s m_p}{r^2}$$

The speed of the planet about the sun (orbital velocity) is:



$$v^2 = \frac{GM_s}{r} \Rightarrow v = \sqrt{\frac{GM_s}{r}}$$

The orbital speed of the planet is

$$v = \frac{2\pi r}{T}$$ where, T is the period of the planet about the sun

$$\frac{(2\pi r)^2}{T^2} = \frac{GM_s}{r}$$

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM_s} = 2.97x10^{-19} s^2/m^3$$

This equation is Kepler's third law: the square of the period is proportional to the cube of the distance of the planet from the sun.

- The proportionality constant K takes the value:

$$K = \frac{4\pi^2}{GM_s} \approx 2.97x10^{-19} s^2/m^3$$

Exercise

Calculate the mass of the Sun, noting that the period of the Earth's orbit around the Sun is 3.156×10^{7} s and its distance from the Sun is 1.496×10^{11} m.

Escape velocity

- ✓ is the minimum speed needed to break free from the gravitational pull of celestial body with out further propulsion.

To scape, total energy (kinetic + potential) must be zero.

$$\frac{1}{2}mv_e{}^2 = \frac{GMm}{r}$$

$$v_e = \sqrt{\frac{2GM}{r}}$$

Where M is mass of celestial body

r distance from the center of the celestial body to the object.

VideoTrimmer_Escape Velocity (1).mp4

Escape velocity is;

- ✓ approximately 11.2 km/s at the surface of the earth.
- ✓ depends on mass & radius of celestial bodies from which the object is escaping.
- ✓ independent of mass of the object.
- ✓ same for objects regardless of their masses.

Notes

- ✓ If object start escaping at higher altitude, escape velocity less than that of when starting from the surface of the celestial body.
- ✓ At them altitude escape velocity & orbital velocity are related as;

$$v_e = \sqrt{2}\, v_o$$

Examples

1. If the escape velocity from the mars is approximately 5 km/s, what would be the escape velocity if mars had the same mass but was half its current radius?
2. If an object has an orbital velocity of 8 km/s, what is the escape velocity?

The end!

Thank you!